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Asset Allocation through Grey Wolf Optimization: A Case of KSE-30 Index

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ABSTRACT

This article is an effort to aid investors by drawing attention to select investment items. Grey Wolf Optimization (GWO), TOPSIS with Eigenvector, Market Capitalization, and the Equal Weighted Technique are the four main methods discussed in this paper. This study uses the KSE-30 Index as its sample size; however, because to a lack of data, only 26 businesses are chosen for analysis using 10 criteria. All four methods are implemented and weights are determined based on these criteria. These weights are then utilized in conjunction with MATLAB's in-built tools to construct a portfolio. Based on its ability to generate the greatest possible portfolio, GWO appears to be a powerful resource for affluent investors. Equal-weighted portfolios performed the worst, followed by the Eigenvector-TOPSIS technique, then Market Capitalization.

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1. Introduction

Main objective of any investor in assets is to earn maximum yield with lowest possible risk. For the very purpose, different techniques have been used to build an effective portfolio. Rezaian, in 2015, stated that careful selection of alternative can produce desired outcomes.

Harry Markowitz in 1952 used the term 'portfolio' for first time in his publication "Portfolio Theory". Hung and Chen (2009); Jahanshahloo, Lotfi, and Izadikhah (2006) describes decision making, a process of evaluating and selecting the best among available alternatives, as it is the key process of investment. Multiple factors play their part and influence decision making process. Hence, various techniques are applied. In MCDM (Multi Criteria Decision Making), external factors and internal factors are judged. These factors impact the organization in various ways whether direct or indirect. Based on these, organization's future wealth and value is forecasted, and the decision is made, as explained by Janani et al, in 2012. Internal aspects includes profitability and liquidity measuring ratios, productivity, management etc., and those of external are technological practices, economic conditions, and social and political aspects (Tiryaki & Ahlatcioglu, 2009). Saremi, Mousavi, and Sanayei (2009), stated these theories and methods purposed by Elton and O'Higgins (2008) to solve MCDM, Elimination Theory (ET), multi-attribute utility theory (MAUT), AHP, PROMETITTEE, ELECTRE and TOPSIS (Saremi et al., 2009).

Rather, Sastry, and Agarwal (2017) described artificial intelligence as a new and effective technique for portfolio selection. ANN is one of the earliest methods among all. Holland (1992) gave the idea of Genetic algorithms, and Soleimani, Golmakani, and Salimi

(2009) also used these optimization techniques that were coming into practice, as Simulated which was used by Crama and Schyns (2003) for complicated portfolio selection. These AI practices spread quickly owing to their effectiveness (Mirjalili, Mirjalili, & Lewis, 2014).

Mirjalili et al. (2014), proposed a Meta heuristic technique identified as Grey Wolf Optimization. It was designed while keeping in mind the imitating behavior of grey wolves. Grey wolves survive by designing a pack, whose hierarchy consists of four levels. Alpha is the leader of the pack, whether male or female, and is the decision maker in all routine chores such as hunting, place to rest, sleeping and waking. They chose alpha on the basis of its management and administration abilities. It has its subordinate, beta, who suggests or take suggestions, then they allocate commands. It is the strongest candidate for Alpha when alpha passes away, then comes deltas who are usually hunters, guards, elders, and caretakers. Last level holds omegas, which are not pointless as they sometimes become a reason to fight between the pack (Mirjalili et al., 2014). Grey wolves portray another captivating quality which is chasing process of prey which consists of three steps. First step is searching for the prey, second is encircling and last is attacking.

AI method, GWO, will be used, in this study, for choosing asset allocation. Other techniques discussed are Market Capitalization technique, equal weights portfolio technique, and Eigenvector-TOPIS technique. A diversified portfolio is created while using all these techniques, represented, and compared in tabular and graphical form.

2. Literature Review

In 1952, Harry Markowitz is acknowledged the pioneers to portfolio selection. Earlier than the introduction of H. Markowitz (1952) portfolio theory, people with low savings used to invest in less riskier stocks as compared to wealthy ones (Hogan, 1994). Hence, H. Markowitz (1952); H. M. Markowitz (1999) provided an out of the box idea as he proposed that investor should assess overall risk and return of portfolio rather than only distinct asset. He introduced the concept mean variance formulation for portfolio selection. Markowitz gave the solution to complex portfolio selection problem as to set a target return then minimizing risks by allocating weights to each asset.

Multi Objective Stochastic Linear Programming (MOSLP) was used by Abdelaziz et al. in 1999. They tried to solve multi criteria decision problems. These criteria include risk, returns and liquidity. It is not possible for a Decision Maker (DM) to discover out all the information, forecasting its happenings based on his or her preferences, which usually based on rough ideas (utility function) (Abdelaziz, Lang, & Nadeau, 1999). Tiryaki and Ahlatcioglu (2009) adopted Fuzzy Analytical Hierarchy Process (AHP), which is useful in uncertain environment. They combined both AHP and multiple criteria approaches for decision making. Technological, Profitability, Economic, Size, Political, and Technological Control were taken as decision-making tools for multiple criteria in their research. They used various methods to calculate weights and then highlighted the better investment options. Huang and Jane (2009) used average autoregressive exogenous (ARX), grey system and rough set (RS) theories, a hybrid system for investment portfolio selections and forecasting stock market trends. They forecasted the bi-annual upcoming data and after that, asset was allocated weights, by using RS on return basis. They discussed Return on Equity, Return on assets, Earnings per Share, equity growth rate, Gross Profit Margin, inventory turnover rate and quick ratio (Huang & Jane, 2009). Tahoori, Fazli, and Kiani Mavi (2011) discussed multiple criteria approach as they used two step methods to check the value of that organization investment decision making process, previous record of stock market and the market success of the organization, and firstly firm's health is grounded on different leverage, liquidity, market value, profitability, and activity ratios. Jerry Ho, Tsai, Tzeng, and Fang (2011) projected a unique method built on MCDM model. Their study states that they take advantage of DEMATEL, VIKOR and ANP to achieve the objective of optimal portfolio solution choice which was based on the CAPM model. Gabus and Fontela (1972) suggested DEMATEL, which is applied to discover out the relationship amongst various criteria. Saaty (1996) introduced ANP to resolve the difficulties associated to response between the criteria. It proceeds by splitting complications into various groups and each group is treated as separate criteria. Ho et al. (2011) used VIKOR and was projected by Opricovic (1998) that is applied to rank the alternatives. Three criteria comprising, expected market return, Risk-Free rate, and beta of security. These criteria impacted with many other factors. As, industrial structure and different other macroeconomic factors have an impact upon expected market return. Financial risk and firm-specific risk affect beta of the security. Risk free rate is dependent upon exchange rate, budget deficit, as well as on discount rate. The results recommended beta of securities as more influential as equated to firm-specific risks and exchange rate.

Various scholars have used Grey Wolf Optimization for different objectives in their studies. Zainal and Mustaffa (2016) predicted gold prices using Grey Wolf Optimizer. Results of techniques, GWO and Artificial neural network were compared, and they ended by stating GWO as a better technique. Later they determined that GWO offers better forecasting than other methods (Zainal & Mustaffa, 2016). GWO is time saving with respect to other techniques which require time and training to take advantage of them. Yusof and Mustaffa (2015) conducted a study to predict oil and energy prices. Results were associated with the findings attained by Artificial Bee Colony and Differential Equation models. Yusof and Mustaffa (2015) also concluded GWO as better approach for predicting prices. It serves not only in financial sector, but Nanda, Gulati, Chauhan, Modi, and Dhaked (2019) employed GWO for Satellite Image Segmentation and was gratified with the results in a shorter time span.

Mazraeh, Daneshvar, and Roodposhti (2022) studied optimizing the stock portfolio of active companies listed on the Tehran Stock Exchange based on the forecasted price. They used a combination of different filtering methods such as optimization of trading rules based on technical analysis (ROC, SMA, EMA, WMA, and MACD at six levels—Very Very Weak (VVW), Very Weak (VW), Weak (W), Strong (S), Very Strong (VS), and Very Very Strong (VVS)), Markov Chains, and Machine Learning (Random Forest and Support Vector Machine) Filter stock exchanges and provide buy signals between 2011 and 2020. Comparison of solution methods shows that the MOGWO (Grey Wolf Optimizer) algorithm has high efficiency in stock portfolio optimization.

When compared to similar population-based algorithms, Grey Wolf Optimizer (GWO) holds its own. Due to its limited global search capabilities, traditional GWO has been replaced by a variant called ADGGWO (X. Yu & Liu, 2022), which employs adaptive Evolutionary Population Dynamics (EPD) strategy, differential perturbation strategy, and greedy selection strategy. In order to optimize the process of task allocation,(Yuvaraj, Karthikeyan, & Praghash, 2021) Yuvaraj et al. (2021) propose using a machine learning model to parallelize the jobs allotted to the event queue and the dispatcher of the serverless framework and make use of the Grey Wolf Optimization (GWO) model. By combining the projection pursuit model (PPM) with the Grey wolf optimization (GWO) technique, S. Yu and Lu (2018) develop a novel integrated PPMGWO model for optimizing the allocation of water resources in a transboundary river basin.

Hasterok et al. (2021) present an optimization model based on the Grey Wolf Optimizer meta-heuristic to enable the definition of ideal energy mix considering the investment and operating expenses, taking into account the power and heating demand projection. Cast-in-place tunnel liners, concrete inner walls, a concrete portal, a concrete ceiling slab, and a concrete slab on grade are the five parts of a highway tunnel that (Abdelkader, Al-Sakkaf, Elshaboury, & Alfalah, 2022) suggest an integrated deterioration prediction model for. The built deterioration model is thought to include two main parts: calibration and evaluation. The first part presents a model for predicting the deterioration of highway tunnel parts that combines Gaussian process regression with the grey wolf optimization method (GWO-GPR).

It is challenging to make broad predictions about returns and hazards when investing in new stocks, as discussed by (Li, Zhou, & Tan, 2022). Therefore, we present a model for optimizing portfolios in the face of a non-deterministic rate of return. Due to the model's complex non-smooth and nonconcave properties, a new version of the grey wolf optimization (GWO) technique is developed. The GWO method outperforms the classic particle swarm optimization algorithm and the genetic algorithm (Li et al., 2022), as demonstrated by the findings.

3. Methodology and Data

A Meta heuristic technique Grey Wolf optimization proposed by (Mirjalili et al., 2014) is used in this paper. According to Mirjalili et al. (2014) Grey Wolf lives by forming packs in the hierarchy of alpha, beta, delta, and omega correspondingly. Most commonly, there are 5-12 members in a pack. But National Wildlife Federation Organization (NWF) shared a report in which they stated that the range can group to 30 wolves. That is why; KSE-30 Index has been designated for the sample. 26 companies are used due to insufficient data availability. From 2009 to 2022 annual reports of companies has been chosen as a panel data. Data is gathered from three main sources: annual statement of companies, state bank of Pakistan website and Pakistan stock exchange webpage.

Selected variables are defined here:

- (Dang, Li, & Yang, 2018) stated that market value, firm assets, and sales can be used measure the company size. Total assets are used here.
- Market Capitalization (MC) is the outstanding shares' market value (Kumar, 2009)
- Price volatility (σ) consisting of the standard deviation of stock prices (Janani, Ehsanifar, & Bakhtiarnezhad, 2012).
- Trading Days (TD) is total number of days stock exchange has traded during the selected period (Janani et al., 2012).
- Trading Quantity (TQ) is the total number of shares traded in Stock exchange on daily basis during the sample period (Janani et al., 2012).
- Current Ratio (CR) (Tahoori et al., 2011).
- Earnings per Share (EPS) (Janani et al., 2012; Tahoori et al., 2011).
- Net Profit Margin (NPM) (Tahoori et al., 2011).
- Return on Assets (ROA) (Tahoori et al., 2011).
- Return on Equity (ROE) (Janani et al., 2012).

3.1. Grey Wolf Optimizer Methodology

GWO is a stimulating and useful optimizer technique used while highlighting the hierarchy level of the data as grey wolves do i. e. in the form of different hierarchy levels alpha(α), beta(β), delta(δ) and omega(ω). It is the modified form of the model used by (Nanda et al., 2019). It will be used to allocate weights of selected sample. Grey wolf's three step chasing process is also used in the paper.

3.2. Algorithm and the Model of GWO

Grey wolves' social behavior is converted in a mathematical form by (Mirjalili et al., 2014) to achieve optimization. Their model is given below:

3.3. Social Hierarchy

All the assets to be considered are arranged in the form of four hierarchy levels as of grey wolves. Alpha is serving as a leader other are subordinates. The fittest combination is alpha, second best is beta, third suitable is delta and the remaining assets are omega. Hence, initial competitors will be alpha, beta and delta, showing the chasing conduct while omega will assist the three.

3.4. Encircling Prey

After searching stalking process, it begins while encircling the prey. That conduct is shown in equation form below:

$$\vec{D} = |\vec{C}.\vec{X}\mathrm{P}(t) - \vec{X}(t)| \quad \& \quad \vec{X}(t+1) = \vec{X}\mathrm{p}(t) - \vec{A}.\vec{D}$$

Here t is the current iteration, $\vec{A}$ and $\vec{D}$ are vectors' coefficient, while$\vec{X}_p$is position prey vector and $\vec{X}$determines position's vector. From top order$\vec{A}$ and $\vec{C}$ can be determined as,

$$\vec{A} = 2\vec{a}.\vec{r}1 - \vec{a} \& \vec{C} = 2.\vec{r}2$$

Changed position of wolves is shown by figure 2 with respect to the prey. They find the best position to prey while updating the positions (X, Y) to (X*, Y*). Another position (X*, -X, Y*) can also be appropriate. 3D effect shows another position (X*, Y*, Z*) as compared to the initial position (X, Y, Z). Positions can be changed by altering the values of $\vec{A}$ and$\vec{C}$. Grey wolves can alter positions inside the given space by using vector $\vec{A}$ and $\vec{C}$.

3.5. Hunting

Most commonly alpha is the leader while chasing but they can change the position as position of the prey is not determined. Best three combinations are still alpha, beta and delta as they are experienced hunters. Omegas pursue those three. They change the position with respect to other three levels.

Mathematical format is given:

$$\vec{D}\alpha = |\vec{C}1.\vec{X}\alpha - \vec{X}|, \vec{D}\beta = |\vec{C}2.\vec{X}\beta - \vec{X}|, \vec{D}\delta = |\vec{C}3.\vec{X}\delta - \vec{X}|$$

$$\vec{X}1 = \vec{X}\alpha - \vec{A}.(\vec{D}\alpha), \vec{X}2 = \vec{X}\beta - \vec{A}.(\vec{D}\beta), \vec{X}3 = \vec{X}\delta - \vec{A}.(\vec{D}\delta)$$

While $\vec{X}(t+1) = \frac{\vec{X}1 + \vec{X}2 + \vec{X}3}{3}$as Figure 1 shows the changing positions of all participants.

3.6. Attacking Prey (exploitation)

Grey wolves attack unless and until prey stops its movement. Its mathematical model shows that value of $\vec{a}$decreases as value of $\vec{A}$ also decreased. It means that$\vec{A}$ valuelies between -2$\vec{a}$ to 2$\vec{a}$ and $\vec{a}$ decrease from 2 to 0 with incremental iterations. As the value of $\vec{A}$ is between -1& 1, the agent alters its position according to the position of prey.

3.7. Search for Prey (exploration)

Grey wolves spread themselves to search the prey, after finding they collectively attack it. This activity is controlled by alpha, beta, and delta. They converge at global level to search for the best position of prey. Vector $\vec{C}$ supports the exploration process. This process pertains local optima and provides best optimization around the globe. The vector$\vec{C}$is presented as an obstaclewhichwill prevent them to approach the prey rapidly and suitably.$\vec{C}$ vector allocates a weight to prey.

3.8. Market Capitalization

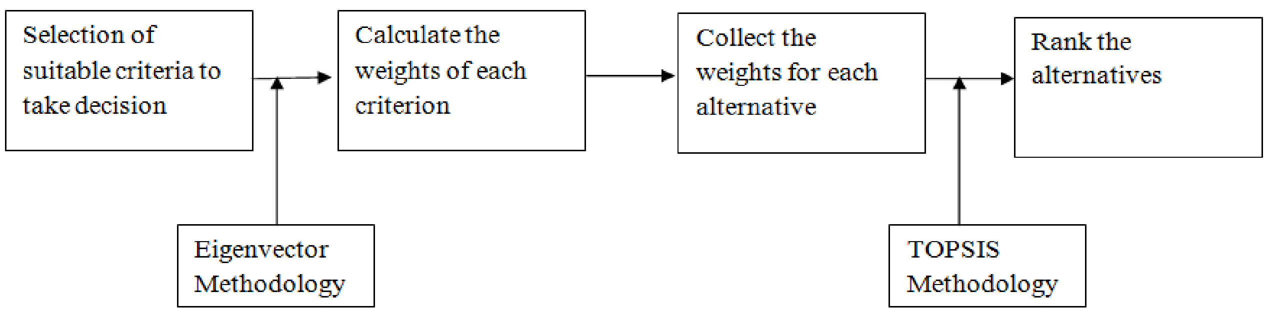
Another method is Market Value weighted method. This approach weights the assets according to market capitalization of assets and allocates weights on the basis of market value with lower transaction cost (Perold, 2007). Hsu (2004) presents its importance. He describes Cap-Weighted as a passive asset allocation technique; it does not demand separate management fee. It modifies itself with varying prices of securities, rebalances weights and relates costs to it. It allocates high weights to large companies due to their higher proportion to stock market as they provide more liquidity, Sharpe portfolio ratio and quicker operations. Market value of each asset is divided by total assets value; hence investor will get a weight. He/she puts savings according to the weights.

$$w_{i,t} = \frac{P_{i,t} * S_{i,t}}{\sum_{k=1}^{N} P_{k,t} * S_{k,t}}$$

3.9. Eigenvector-TOPSIS

The Eigenvector – TOPSIS Methodology ranks the companies to identify the top organization for investment. To identify that organization, multi criteria is used.
Steps are:

- choosing suitable criteria and presenting the alternatives available.
- Applying Eigenvector methodology for measuring the weights of each criterion.
- Collecting the weights for each alternative.
- Applying TOPSIS method for ranking and selecting the best.



The methodology is described here:

Variables are criteria and companies are alternatives in the company, to highlight the best one.

Saaty (1996) presented Eigenvector model to resolve problems involving multi criteria decision making process. It generates and measures weights after evaluating and comparing all alternatives. SPSS software has been utilized for that purpose, where factor analysis provided ground for Eigenvector value and used as weights.

Hwang, Yoon, Hwang, and Yoon (1981) presented TOPSIS model which ranks alternatives by maximizing and minimizing weights from an ideal point. It runs on the assumption that optimal alternative holds minimum distance from positive solution and from negative it has maximum distance Hwang et al. (1981); Janani et al. (2012).

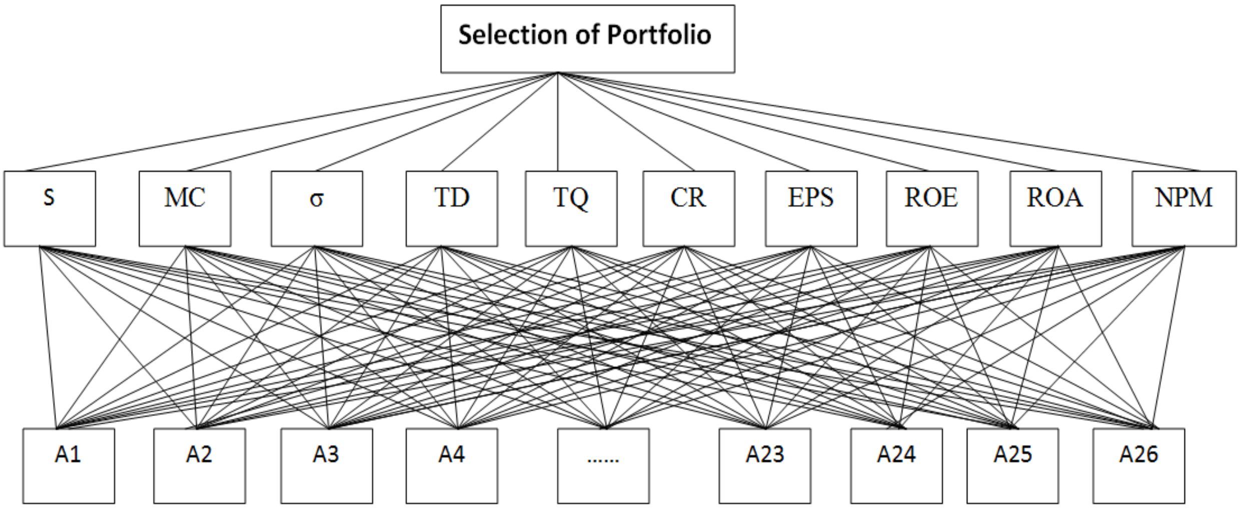


Figure: 1 Hierarchical model used in this study

'n' alternatives are representing input while m is denoting criteria, shown in the form of a matrix for TOPSIS method.

$$X = \begin{bmatrix} x_{11} & x_{12} \cdots & x_{1m} \\ x_{21} & x_{22} \cdots & x_{2m} \\ \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} \cdots & x_{nm} \end{bmatrix}$$

Stepwise TOPSIS methodology is given by:

Step No. 1: A matrix containing normalized values is formed and normalized decision matrix 'R' is computed, given as 'X' above.

Step No.2: Normalized weighted matrix is computed when the weights are multiplied 'R' which forms normalized decision matrix.

Step No.3: Computation of the positive ideal point 'A^+' and negative ideal point 'A^-' is followed by Normalized weight matrix given as,

$$A+ = \{V+1, \dots, V+n\} = \{(max\, Vij \mid i\, \varepsilon\, I), (min\, Vij \mid i\, \varepsilon\, J)\}$$ and

$$A- = \{V-1, \dots, V-n\} = \{(max\, Vij \mid i\, \varepsilon\, I), (min\, Vij \mid i\, \varepsilon\, J)\}$$

Where 'I' is favored criteria and 'J' means cost criteria.

Step No.4: Positive & Negative Distance d^+_i and d^-_i is estimated as,

Positive distance is $d_i^+ = \left\{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2\right\}^{1/2}, i = 1, \dots, m$ and

Negative distance is $d_i^- = \left\{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2\right\}^{1/2}, i = 1, \dots, m$

Step No. 5: Relative closeness to the ideal point is computed as,

$$CL = \frac{d_i^-}{d_i^- + d_i^+}, CL \epsilon [0,1]$$

Step No. 6: Alternatives are ranked.

Weights are assigned based on ranking, for 26 alternatives we divided into 3 different groups. First group has 50% weightage, second has 30% and last has 20% weightage. First two groups contain 9,9 alternatives while last has 8. It means that top 9 companies have 50% weightage. Portalloc and portopt, built in functions of MATLAB are applied.

Portalloc function is employed for computing Port risk (Standard deviation), Port Return (Expected returns), PortWts (weights assigned to each asset) of all risky assets. Weights sum must be zero, riskless rate (KIBOR or lending rate), Borrow Rate (optional borrowing rate either in decimal or Nan (default)), and Risk Aversion (investor's risk aversion attitude, avg. value is 3 and highest means intensity of risk aversion).

Portopt is used for mean-variance portfolio optimization. Net portfolio returns mean return proxy, variance in portfolio returns indicates risk proxy, and portfolio set as controversies. Exp Returns, Exp Covariance (among the assets), Num Ports (number of portfolios generated by the function), Port Return, Conset (default, optional and referred to constraints matrix).

These two MATLAB functions helps up find optimal weights of assets and their investment weights are calculated for the formulation of optimal portfolios. Minimum investment of rupee '1' is supposed for the purpose of optimal portfolio computation.

4. Results and Discussion

Eigenvector-TOPSIS methodology is applied to rank the assets. IBM SPSS Statistics 25 is used to calculate the values of Eigen. The weights of criteria used in study are:

Table 1: Weights Calculated through Eigenvector Methodology

NPM	ROA	ROE	Size	CR	EPS	TQ	σ	TD	MC
0.23074	0.27926	-0.00088	0.0694	0.0388	0.17047	-0.09586	0.115019	0.011052	0.18194

ROE and Trading Quantity has negative weights. Highest weight is held by Return on Asset.

Normalized weights matrix for TOPSIS is found with respect to these weights:

Table: 2 Normalized Decision Matrix

Normalize Data										
	NPM'	**ROA'**	**ROE'**	**Size'**	**CR'**	**EPS'**	**TQ'**	**'σ'**	**TD'**	**MC'**
OGDC'	0.196	0.082	0.006	4.147	2.023	2.786	3.932	2.201	1.088	4.60
PPL'	0.158	0.071	0.005	3.850	0.713	4.675	3.837	3.234	1.085	4.12
POL'	0.169	0.072	0.008	3.344	0.532	15.949	3.610	12.978	1.078	3.73
NBP'	0.029	0.0004	4.287	4.770	2.297	0.696	4.192	0.356	1.074	3.88
HBL'	0.025	0.0002	3.686	4.569	1.259	1.136	3.493	3.078	1.071	4.05
UBL'	0.053	0.0006	2.129	4.61	1.525	1.991	3.865	2.217	1.081	3.89
MCB'	0.095	0.002	1.772	4.485	1.416	3.509	3.673	3.327	1.078	4.08
BAHL'	0.021	0.0002	1.432	4.265	1.062	0.258	3.473	0.076	1.077	3.41
FCCL'	0.027	0.013	0.002	3.117	0.095	0.084	4.320	0.043	1.077	3.04
MLCF'	0.005	0.004	0.0001	3.175	0.058	0.068	4.130	0.351	1.079	2.89
DGKC'	0.034	0.008	0.0006	3.447	0.389	1.087	4.351	1.431	1.077	3.28
LUCK'	0.0355	0.029	0.003	3.472	0.264	8.428	3.775	17.958	1.077	3.67
FFC'	0.020	0.038	0.019	3.518	0.078	1.493	3.963	0.894	1.076	3.80
ENGRO'	0.016	0.008	0.002	3.886	0.132	6.873	4.161	5.116	1.076	3.71
HUBC'	0.006	0.006	0.005	3.742	0.083	0.468	3.935	0.268	1.089	3.63
KAPCO'	0.009	0.013	0.006	3.551	0.107	0.618	3.499	0.381	1.077	3.48

SNGP'	0.0001	0.0002	0.0004	3.861	0.051	0.189	3.901	0.386	1.088	3.26
PSO'	0.0001	0.002	0.002	4.027	0.091	15.545	3.788	8.746	1.071	3.60
EPCL'	0.000	0.0002	0.000	3.077	0.046	0.019	3.981	0.039	1.074	3.08
TRG'	0.019	0.089	0.089	2.784	0.028	0.153	4.503	0.088	1.074	2.58
PAEL'	0.002	0.002	0.0001	3.156	0.243	0.014	3.990	0.323	1.089	2.76
NML'	0.009	0.005	0.0005	3.469	0.119	0.393	4.046	0.975	1.077	3.31
SEARL'	0.016	0.031	0.004	2.605	0.242	1.489	3.098	11.336	1.075	2.84
ATRL'	0.000	0.0002	0.0001	3.486	0.066	0.662	3.130	11.177	1.077	2.94
NRL'	0.002	0.009	0.002	3.354	0.212	19.075	3.071	16.092	1.072	3.15
MTL'	0.014	0.091	0.016	2.797	0.235	41.088	2.635	44.127	1.077	3.18

Matrix R is calculated by multiplying weights (calculated by Eigenvector values) with the normalized matrix. That is weighted normalized matrix:

Table: 3 Weighted Normalized Decision Matrix

	NPM'	ROA'	ROE'	Size'	CR'	EPS'	TQ'	'σ'	TD'	MC'
OGDC'	0.05	0.0231	0.0000	0.2881	0.0791	0.46914	-0.37	0.26	0.0132	0.83
PPL'	0.07	0.0201	0.0001	0.2671	0.0311	0.79712	-0.36	0.37	0.0132	0.75
POL'	0.09	0.0201	0.0002	0.2322	0.0211	2.71121	-0.34	1.49	0.0132	0.68
NBP'	0.07	0.0001	-0.0041	0.3311	0.0891	0.10922	-0.40	0.04	0.0132	0.71
HBL'	0.06	0.0001	-0.0031	0.3172	0.0492	0.19222	-0.33	0.35	0.0132	0.73
UBL'	0.02	0.0000	-0.0021	0.3223	0.0592	0.34124	-0.37	0.25	0.0132	0.72
MCB'	0.02	0.0003	-0.0021	0.3115	0.0551	0.60412	-0.35	0.38	0.0132	0.75
BAHL'	0.05	0.0001	-0.0013	0.2965	0.0412	0.04921	-0.33	0.02	0.0132	0.62
FCCL'	0.006	0.0031	0.0001	0.2161	0.0042	0.00521	-0.41	0.01	0.0132	0.55
MLCF'	0.0001	0.0011	0.0001	0.2223	0.0021	0.01322	-0.39	0.05	0.0132	0.53
DGKC'	0.0008	0.0021	0.0001	0.2391	0.0155	0.18522	-0.41	0.16	0.0132	0.59
LUCK'	0.0008	0.0081	0.0002	0.2412	0.0123	1.43223	-0.36	2.06	0.0132	0.67
FFC'	0.004	0.0112	0.0000	0.2442	0.0031	0.2542	-0.38	0.11	0.0132	0.69
ENGRO'	0.005	0.0021	0.0000	0.2713	0.0052	1.1642	-0.39	0.58	0.0132	0.68
HUBC'	0.0011	0.0022	0.0001	0.2612	0.0031	0.0762	-0.37	0.03	0.0132	0.67
KAPCO'	0.0021	0.0041	0.0001	0.2462	0.0051	0.1042	-0.33	0.04	0.0132	0.64
SNGP'	0.0000	0.0001	0.0001	0.2682	0.0022	0.0252	-0.37	0.04	0.0132	0.59
PSO'	0.0001	0.0000	0.0001	0.2791	0.0043	2.6522	-0.36	1.02	0.0132	0.66
EPCL'	0.0002	0.0000	0.0002	0.2142	0.0021	0.00022	-0.38	0.02	0.0132	0.57
TRG'	0.0004	0.0252	0.0000	0.1931	0.0012	0.02822	-0.43	0.02	0.0132	0.48
PAEL'	0.0000	0.0000	0.0000	0.2192	0.0091	0.00522	-0.38	0.04	0.0132	0.52
NML'	0.0020	0.0011	0.0001	0.2413	0.0052	0.0562	-0.38	0.11	0.0132	0.61
SEARL'	0.0040	0.0081	0.0001	0.1812	0.0091	0.2522	-0.29	1.30	0.0132	0.52
ATRL'	0.0000	0.0000	0.0000	0.2422	0.0032	0.1192	-0.32	1.28	0.0132	0.53
NRL'	0.0001	0.0030	0.0000	0.2331	0.0081	3.2452	-0.29	1.85	0.0132	0.58
MTL'	0.0031	0.0250	0.0002	0.1941	0.0091	6.9932	-0.25	5.07	0.0132	0.59

Each of the criteria has these positive and negative ideal point values:

Table: 4 Positive ideal Point Values

	NPM'	ROA'	ROE'	Size'	CR'	EPS'	TQ'	'σ'	TD'	MC'
'V^+_j'	0.047	0.0253	-0.0000000049	0.332	0.0892	6.993	-0.256	0.00459	0.0119	0.837

Table: 5 Negative ideal point values

	NPM'	ROA'	ROE'	Size'	CR'	EPS'	TQ'	'σ'	TD'	MC'
'V^-_j'	0.00000124	0.0000479	-0.0038	0.1812	0.02157	0.00419	-0.4417	5.0747	0.0128	0.482

Table: 6 "Calculation of d+i , d-i , d+i + d-i , CLi and Ranking priority"

Companies	'$d^+_{I'}$	'$d^-_{I'}$	'$d^+_i + d^-_{I'}$	'CL'	'Priority Ranking'
OGDC'	6.5364	4.8605	11.3952	0.42	8
PPL'	6.2049	4.7795	10.9884	0.43	6
POL'	4.5158	4.4991	9.0389	0.49	4
NBP'	6.8811	5.0439	11.9106	0.42	11
HBL'	6.8195	4.7359	11.5456	0.41	23
UBL'	6.6522	4.8414	11.5028	0.42	14
MCB'	6.4052	4.7419	11.1465	0.44	7
BAHL'	6.9463	5.0711	12.0913	0.42	12
FCCL'	6.9992	5.0712	12.0818	0.42	16
MLCF'	6.9809	5.0351	12.0469	0.41	20
DGKC'	6.8191	4.9156	11.7214	0.42	24

LUCK'	5.9371	3.3408	9.2082	0.36	19
FFC'	6.7411	4.9836	11.7711	0.42	9
ENGRO'	5.8602	4.6406	10.5367	0.44	5
HUBC'	6.9272	5.0499	11.9081	0.42	13
KAPCO'	6.8955	5.0358	11.9893	0.46	10
SNGP'	6.9785	5.0336	12.0721	0.41	18
PSO'	4.4671	4.8607	9.3793	0.52	3
EPCL'	7.0061	5.0716	12.0224	0.43	15
TRG'	6.9703	5.0649	12.0339	0.42	17
PAEL'	6.9937	5.0381	12.0617	0.42	22
NML'	6.9423	4.9652	11.9009	0.41	21
SEARL'	6.8709	3.7823	10.6537	0.35	25
ATRL'	7.0007	3.7941	10.7459	0.35	26
NRL'	4.1891	4.5781	8.7136	0.52	2
MTL'	5.0767	6.9954	12.751	0.57	1

Highest ranking is pertained by MTL (Millat Tractor Limited), then NRL (National Refinery Limited).

Share price and number of shares outstanding are multiplied to calculate market capitalization. Its 10 years avg. market capitalization is used in computation of weights which are further utilized to generate optimal portfolio.

First column has information regarding the sample. Second has ten years avg. market capitalization. Third has weights, last row has total of all variables. Highest weight i.e., of 25% is held by OGDCL (oil and gas Development Company limited), second is PPL (Pakistan Petroleum Limited) containing 8.74%weight. These are used for computation of optimal portfolio.

In Grey Wolf optimization the upper and lower bound limit are used as input 200 iterations and 26 alternatives or grey wolves. Optimal positions of grey wolves have been computed using MATLAB. After putting upper and lower bound values as input, it provides the 26 best positions for each alternative.

AS GWO provides optimal values, so I normalize these values using MATLAB software. All the optimal values are multiplied with the companies' returns. Derived matrix is used to highlight the weights of the companies. Results are shown in graphs presenting market capitalization as containing highest volatility, which is relatively low in other methods.

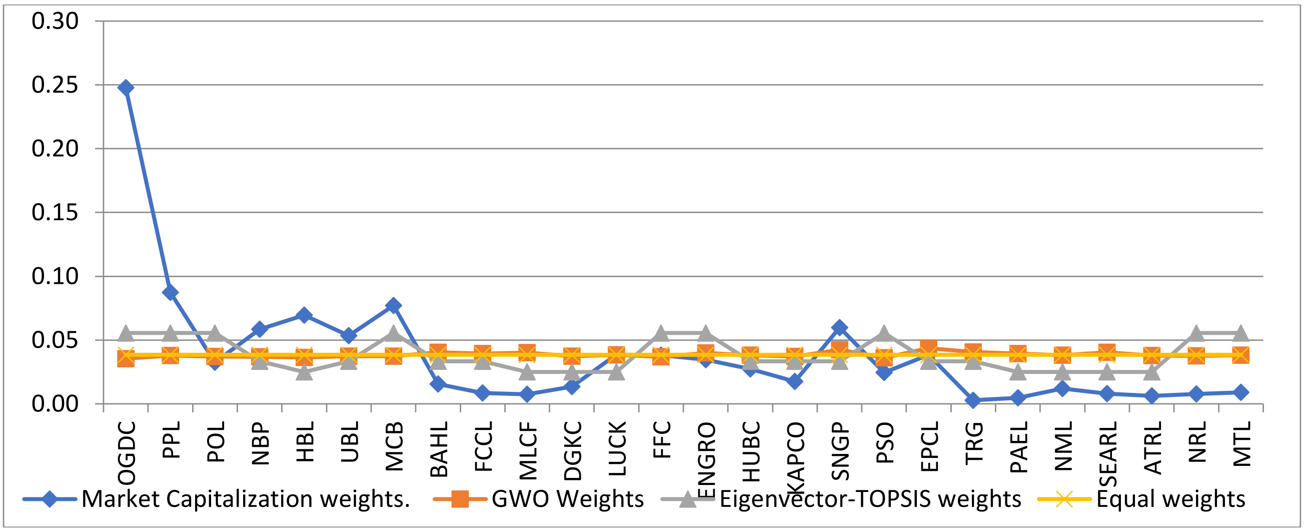


Figure: 3 Weights Generated by the four Techniques

Now weights are entered in MATLAB portfolio optimization functions. Portalloc and Portopt are used to calculate wealth of the investor. Minimum investment is supposed of rupee 1:

Table: 7 Accumulated Wealth of the four techniques. T-test has been applied to check significance level and that shows all the values are significant

Investment Portfolio	MC	EW	GWO	TOPSIS
1st	1.5287	1.5287	1.4181	1.4791
2nd	1.7405	1.7448	1.5375	1.6441
3rd	1.6214	1.554	1.2803	1.486
4th	1.9359	1.8784	1.6612	1.8213
5th	2.2058	2.1575	2.0623	2.136
6th	2.4992	2.458	2.4907	2.4993
7th	2.3421	2.2252	2.6593	2.6058
8th	2.8709	2.7992	3.3628	3.1918
9th	2.6259	2.5603	2.8013	2.7687
10th	2.4877	2.4256	2.6132	2.6028
T-test Significance	0.00	0.00	0.00	0.00

Number of portfolios are 10, wealth of the investor is shown accordingly. Grey Wolf provides Rs. 2.6132 for only Rs. 1 as investment. Second-best option is Eigenvector-TOPSIS, which is offering Rs. 2.6028 to investor. On third number there is Market Capitalization strategy giving Rs. 2.4877 and last is equal weights which gives Rs. 2. 4256. Results suggest GWO as best strategy. To accept the hypothesis a sample t-test is utilized to verify the significance level. Based on t-test, hypothesis is accepted meaning that it is true and providing best arrangement of assets.

Conclusion

This study is conducted to identify the best assets for allocation to investors. For a decision maker it can be very much difficult to identify and collect all the information about decision making, but a decision maker can identify some specific attributes for its decision-making process and preferences of those attributes also have some possible consequences on the mind of decision makers. There are different assets available for investors to invest and classify these into different groups based on some already defined characteristics. These criteria or attributes for an investor can be in different forms, it can be internal to organization factors as well as external factors.

Four different techniques used in this study including, GWO, Eigen value-TOPSIS, Market Capitalization and Equal weights. It is one of the first studies in Pakistan to compare these four techniques at the same time. While GWO is also first time proposed for portfolio selection in Pakistan as the evidence isn't available so far. Same can be saying about Eigenvector-TOPSIS methodology. Although Eigenvector-TOPSIS has been used in many studies, but it was just related to the ranking of assets (Amiri, Zandieh, Vahdani, Soltani, & Roshanaei, 2010; Janani et al., 2012), in this study those ranking converted into weights and these weights later used to create portfolio. For portfolio creation MATLAB software is used. In MATLAB two built-in functions "portopt" and "Portalloc" are used in this regard. Ten portfolios are created based on the weights given by the above mentioned four techniques. For experiment Rs. 1 was invested in the portfolio and after 10 portfolios the amount was checked. The findings indicate that Grey Wolf is increasing asset allocation significantly for the investors and it is one of the better options for the purpose of investment. Eigenvector-TOPSIS is an easy technique which an investor can use for portfolio selection. The results show that Eigenvector-TOPSIS is the second-best technique which is used in this study followed by Market Capitalization and in the last it is equal weights.

Apart from stocks, there are number of other securities available for investors like bonds, foreign exchange markets, Forex Exchanges, Treasury bills etc., by creating more diversified portfolio the risk can be mitigate. External factors like technology, economy, political behavior, inflation, trading system, trading tool, technical analyst etc. can also be included in future study.

Grey Wolf Optimizer is relatively new technique, which is used in this study, there are also number of other techniques available which are yet to adopt in the process of Portfolio Optimization just like, Krill Herd (KH), Gravitational search (GS), Roach Infestation Optimization (RIO), so there is a window open for the new studies for many other people in this regard.

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