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Integrating the Relationship Between Exponents of the Utility Function and the Proportion of the Substitution and Income Effects of the Slutsky Equation into the Microeconomics Literature

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ABSTRACT

The primary purpose of this paper is to explore the relationship between exponents of a linearly homogeneous (Cobb-Douglass type $U = x_1^{\alpha} x_2^{\beta}$) utility function and magnitude of the substitution effect (SE) and income effect (IE) in both the Slutsky and Hicks approaches. It is explored that when the price of a good with a larger exponent in the utility function increases the income effect is greater than the substitution effect, and the converse holds when the price of a good with a relatively smaller exponent increase. When exponents of both goods in the utility function are of the same magnitude, the income and substitution effects are equal irrespective of whether the price of the good $x_1$or good x_2 changes. This concept has not been explored before and thus, it should be included in Microeconomics literature. It has also been shown that for a given utility function the ratios of the utility elasticities of the goods equal the ratio of the income and substitution effects. Also, the Marshallian demand curve is more price elastic than the Slutsky demand curve which is more elastic than the Hicks demand curve.

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1. Introduction

The usefulness of the income effect and substitution effect cannot be overstated. The following examples will elucidate the importance of the income and substitution effect. Knowledge of these two effects is used in consumer choice theory to understand the behavior of buyers and their reactions to price changes. The compensation variation (CV) and equivalent variations (EV) are two measures used to measure welfare changes due to changes in the prices of goods. A decrease in the price of a good causes a positive economic change and would change both CV and EV. It is also instrumental and very useful in firms' decision-making process because changes in the prices of goods are directly related to the firm's decision-making process because changes in price change the commodity's demand and its related complementary and substitute goods. Understanding SE and IE helps us to analyze the relationship between different goods, which is important for studying the relationship between different goods. For example, when demand for a certain particular good increase due to a fall in its price, demand for its complementary good will also increase. These changes will be considered when advertising strategies are designed and implemented. A successful advertising strategy will shift the demand curve to the right and make it more inelastic. Changes in the elasticity of demand will enable us to know the incidence of an indirect tax on buyers and sellers.

The distribution of the tax incidence between the buyer and the seller depends on the elasticity of demand and supply of the commodity in question. Who will pay how much depends on their respective elasticity of demand. If demand is relatively more inelastic than supply, the buyer will pay most of the indirect tax. In the subsidy case, the buyer will receive most of the subsidy if demand is more inelastic than supply.

In consumer choice theory, the total effect of a change in the price of a good on the quantity demanded is broken down into the substitution effect and the income effect by two famous economists Hicks (British) and Slutsky (Russian). To do what is needed, it is assumed that a consumer is consuming two goods x_1 and x_2 with p_1 and p_2 at their respective prices (Amin & Dogan, 2021; Amin et al., 2024). When p_1 decreases from £8 to £4 some income will be freed up if he retains the initial bundle of the goods, because of this fall in the price he can spend more on both goods. This is called the Hicks substitution effect shown by moving from F to F_1 in Figure 1 and the effect due to the freed-up income is called the Hicks income effect, demonstrated by moving from F_1 to G in Figure 1.

In the Slutsky approach (Figure 2), the consumer's income is decreased in response to the decrease in the price which a new dotted budget line KL shows (Figure 2) passing through the initial consumption bundle, the slope determined by the new prices and the consumer's optimal choice at point B on this budget line but on a higher indifference curve U_1 in Figure 2. The idea is that the consumer is given just enough money to purchase an initial bundle of both goods at the new prices. Moving from A to B in Figure 2 is called the Slutsky substitution effect; while moving from B to C in Figure 2 is called the Slutsky income effect. The sum of the two effects is termed the total effect of the price change represented by moving from A to C in Figure 2. It is clear that the Slutsky substitution effect (3.13) is bigger than the Hicks substitution effect (2.58), but the Slutsky income effect (3.13) is smaller than the Hicks income effect (3.67) (Figure 1 and 2 respectively). The Slutsky demand curve is more elastic than the Hicks demand curve.

Calculations obtained for substitution and income effect using the Slutsky approach are more exact than the Hicks approach. The Slutsky approach is considered to be superior to the Hicks approach because the former approach leads to a higher indifference curve where the consumer is achieving greater satisfaction (Figure 2).

2. Literature Review

There is not enough literature available on the relationship between the exponents of the utility function and the magnitude of the substation effect (SE) and income effect (IE) in the Slutsky equation in consumer theory (Varian, 1992). Thaver (2013) published a paper titled Integrating the Output and Substitution Effects of Production into the Intermediate Microeconomics Textbook parallel to an indifference curve analysis to explain the income and substitution effects of a change in the price of a good x on the demand for it. In line with that, it is the contention in this paper that integrating these effects into advanced microeconomics textbooks will service the discipline (Amin, Altinoz, & Dogan, 2020; Amin, Dogan, & Khan, 2020). This analysis is intended to enable an evaluation of public policies designed to increase consumer welfare (Varian, 2014; Varian, 2020). The relationship between exponents of the utility function and the magnitude of SE and IE has not been investigated before.

2.1. The Magnitude of Substitution and Income Effects of a Change in The Price of A Normal Good

To compare the magnitude of the substitution effect and income effect of the price change of a normal good, we assume that (1) the buyer's utility function is linearly homogeneous Cobb-Douglas type $U = x_1^{\alpha} x_2^{\beta} where \alpha + \beta = 1$; (2) the elasticity of substitution between the goods is unity; (3) the price p_1 of good x_1 decreases while the price p_2 of good x_2 remains constant; (4) the indifference curves are concave from above.

3. Methodology

To obtain the desired results simple calculus is used. Math Type and WordPerfect X4 software were used to write equations with mathematical symbols and draw figures. The following example proves our point.

Given the three linearly homogeneous Cobb-Douglas utility functions: i. $U = x_1^{0.5} x^{0.5}$, ii. $U = x_1^{0.3} x^{0.7}$ and iii. $U = x_1^{0.7} x^{0.3}$ subject to the budget constraint $M = p_1 x_1 + p_2 x_2$. Given M= £100, initially, p_1=£8, and p_2= £2, then p_1 decreases to £4. Calculate the respective SE and IE for each of them and show that for i. SE=IE, ii. SE > IE and iii. IE > SE.

Solution

For $U = x_1^{0.5} x^{0.5}$, the Marshallian demand functions are $x_1 = \frac{0.5M)}{p_1} and x_2 = \frac{0.5M}{p_2}$ (1)

The Initial bundle of goods at F (Fig.1) is $x_1 = \frac{0.5(100)}{8} = 6.25 and x_2 = \frac{0.5(100)}{2} = 25$

Expenditure on x_1 and x_2 is $\frac{p_1 x_1}{M} = 0.5(exponent of x_1) and x_2 = \frac{p_2 x_2}{M} 0.5(exp o nent of x_2)$ respectively. $U_0 = (6.25)^{0.5}(25)^{0.5} = 12.5(Figure 1 and 2)$. The final bundle of goods after falling in p_1 from £8 to £4 (=p_1^*) is $x_1 = \frac{0.5(100)}{4} = 12.5 and x_2 = \frac{0.5(100)}{2} = 25$ (Figure 1 at G and Figure 2 at C). The utility level is $U_2 = (12.5)^{0.5}(25)^{0.5} = 17.67(Figure 1 and 2)$.

The utility maximization condition is $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2} \Rightarrow \frac{o.5 x_1^{-0.5} x_2^{0.5}}{4} = \frac{o.5 x_1^{0.5} x_2^{-0.5}}{2} \Rightarrow x_2 = 2x_1$

Substituting this in the utility function $U = x_1^{0.5} x^{0.5} \Rightarrow 12.5 = x_1^{0.5}(2x_1)^{0.5} \Rightarrow$

$x_1 = 8.83, x_2 = 17.67$. The Hicks Demand functions are $x_1^H = U\sqrt{\frac{p_2}{p_1^*}} and x_2^H = U\sqrt{\frac{p_1^*}{p_2}}$ (2)

Plugging in the respective values, we get $x_1^H = 12.5\sqrt{\frac{2}{4}} = 8.83 and x_2^H = 12.5\sqrt{\frac{4}{2}} = 17.67$

Thus, Hick's demand functions at tangency point F1 in Figure 1 are (8.83, 17.67).

The Slutsky demand functions are $x_1^S = \frac{0.5M^*}{p_1^*} and, x_2^S = \frac{0.5M^*}{p_2}$ (3)

Plugging in the respective values we get the Slutsky bundle $x_1^S = \frac{0.5(75)}{4} = 9.37, x_2^S = \frac{0.5(75)}{2} = 18.75$. at B Figure 2. It is to be noted that $M^*$is the minimum income required to buy the initial bundle at a decreased price of x_1 which is denoted by p_1^*=£4. M^*= $p_1^* x_1 + p_2 x_2$= 4(6.25) +2(25) =75.The utility level at the Slutsky bundle is

U_1= $(9.375)^{0.5}(18.75)^{0.5}$= 13.25 Figure 2. Thus, the TE of decrease in p_1 on the quantity demanded of good x_1 (Figure 2) equals C-A= 12.50-6.25= 6.25 units of good x_1. The proportion of SE is B-A= 9.37 units and the remaining units are due to IE (C-B) =12.50-9.37= 3.13.

TE= SE (3.12) +IE (3.13) = 6.25. It is to be noted that the SE equals the IE irrespective of whether the price of x_1 or x_2 changes when expenditure incurred on each good is the same.

The utility function $U = x_1^{0.5}x^{0.5}$, holds the following property.

$$\frac{\%expenditureonx_1}{\%expenditureonx_2} = \frac{utilityelaticityofx_1}{utilityelasticityofx_2} = \frac{exponentofx_1(0.5)}{exp\,o\,nentofx_2(0.5)} = \frac{IE(3.13)}{SE(3.13)} = 1$$
$$IE = SE$$

An increase in consumption of good $x_1(\Delta x_1)$ due to the Hicks SE and IE is 2.58 and 3.67 respectively (Figure 1). An increase in good $x_1(\Delta x_1)$ consumption due to the Slutsky SE and IE is 3.2 and 3.13 respectively (Figure 2). The magnitude of SE and IE are directly related to the proportion of income spent on each good which is represented by the exponents of the goods in the given utility function.

3.1. A Graphical Exposition of the S.E. and I.E. Effects of the Price Fall of a Normal Good

According to Hicks, the SE is shown by the change in quantity demanded of good x_1 because of a change in ratio$\frac{p_{x_1}}{p_{x_2}}$, leaving the consumer on his initial indifference curve U_0 (moving from F to F_1= mn, Figure 1). It is shown by a downward parallel shift of the AB iso-cost curve to DE (Figure 1). The new equilibrium point is obtained at F_1 where DE is tangent to the indifference curve at F_1. The movement from F to F_1 distance mn, is the SE owing to a decrease in the relative price of good x_1, holding expenditure constant. It is to be noted that the consumer remains at the initial indifference curve U_0. Negative cross elasticity of substitution between the price of x_1 and quantity of x_2 causes reduced consumption of x_2 (distance ac).

The IE is a change in the demand for a good due to a change in a consumer's purchasing power, which is, due to a change in their real income. It is shown by drawing a parallel to the DE iso-cost curve AB in Figure 1, leaving the new price ratio unchanged at $\frac{p^*_{x_1}}{p_2}$. This shift in the iso-cost curve reveals the available money to buy more x_1 and x_2. The final equilibrium is at G where the iso-cost AB is tangent to U_1. The movement from F_1 on indifference curve U_o (distance np) to G on indifference curve U_2 is the income effect of a decrease in the relative price of x_1, *ceteris paribus*. Since both x_1 and x_2 are normal goods, the consumer's additional expenditure power spurs it to consume more x_1, as shown by distance np, and more x_2 as shown by distance cb in Figure 1. Put differently, the IE effect is obtained by subtracting the SE from the total effect (TE). IE= TE-SE

The TE of a reduction in p_1 is shown by moving from F to G or horizontal distance mp in Figure 1. where the SE (mn) and IE (np) reinforce each other. It is to be noted that when p_1 decreases; SE= IE if the expenditure incurred on each good is equal i.e. exponents of the utility function $U = x_1^{0.5}x_2^{0.5}$from which the demand function is derived are equal in magnitude. SE > IE if the price of the good decreases on which lesser expenditure is incurred, it is good x_1 in the utility function $U = x_1^{0.3}x_2^{0.7}$. IE > SE if the price of the good decreases on which a greater proportion of income is spent such as in the utility function $U = x_1^{0.7}x_2^{0.3}$. This relationship between utility function exponents and the comparison of magnitudes of SE and IE has not been explored before.

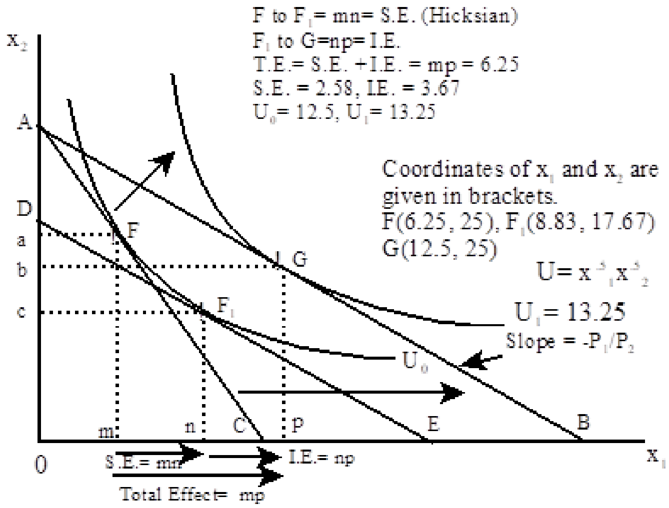

Figure 1: Hicks Substitution Effect and Income Effect

$TE = SE(2.58) + IE(3.67) = 6.25(Hicksapproach)$

In the case of an L-shaped Leontief-type indifference curve, the consumer will be using a fixed proportion of both goods, there would be no substitution between the goods because of the complementarity between good x_1 and x_2. Thus, the indifference curve in Figure 1 will be right-angle at equilibrium F and G, and there will be no substitution effect of price change. Any decrease in the relative price x_1 will result in only IE due to the consumer's increased expenditure power. After the decline in p_1 the consumer will move to a higher indifference curve, he will consume more of both goods x_1 and x_2 in a fixed proportion. Thus, as the slope of an indifference curve at equilibrium increases, SE decreases and IE increases and vice versa.

In the Hicks approach (Figure 1), the IE due to change in its price is removed by returning the consumer to the same level of utility as before the price change. In the Slutsky approach (Figure 2), the consumer is returned to the same quantity of commodity purchased as before the change (Figure 2). In this approach, the consumer has sufficient income to purchase his original bundle at A. The consumer moving to a higher indifference curve means more satisfaction, whereas, in the Hicks approach, the consumer does not move to a higher indifference curve which means the same satisfaction. It can be easily proved that the Hicks demand curve is steeper than the Slutsky demand curve. Put differently, the Slutsky demand curve is more price elastic than the Hicks demand curve.

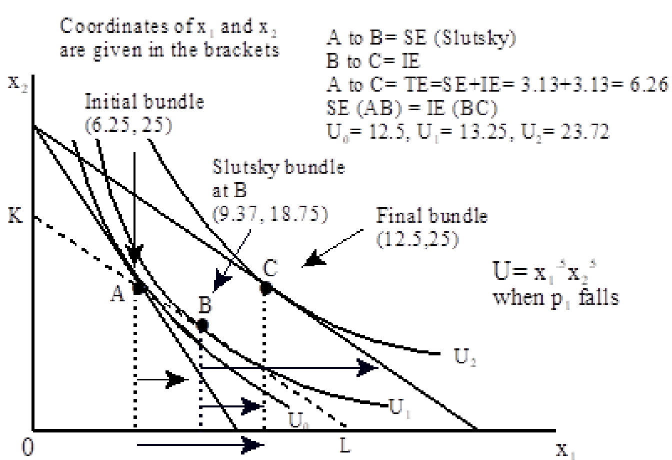

Figure 2: Slutsky Substitution Effect and Income Effect

$$\frac{\%expenditureongood1}{\%expenditureongood2} = \frac{utilityelasticityof x_1}{utilityelasticityof x_2} = \frac{Exponentof x_1(.5)}{Exponentof x_2(.5)} = \frac{IE(3.13)}{SE(3.13)} = 1$$
$$\Rightarrow IE = SE., TE. = SE(3.13) + IE(3.13) = 6.26(Slutskyapproach)$$

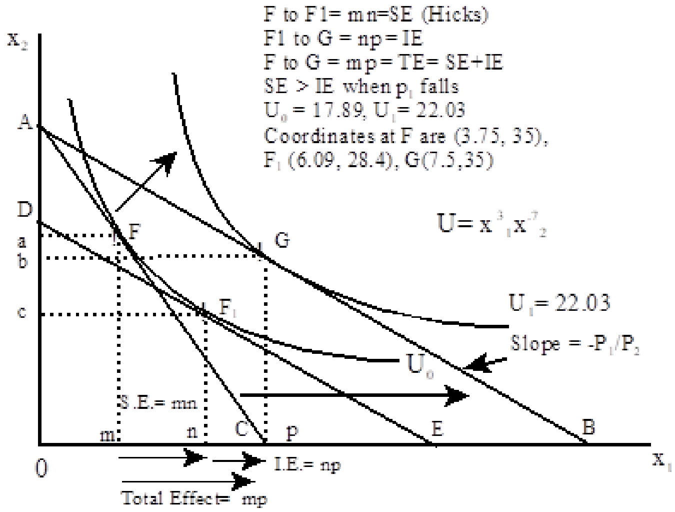

Figure 3: Hicksian Substitution Effect and Income Effect

For $U = x_1^{0.3}x^{0.7}$, the Marshallian demand functions are $x_1 = \frac{0.3M}{p_1} and x_2 = \frac{0.7M}{p_2}$ (4)
The Initial bundle of goods at F (Fig.3) is $x_1 = \frac{0.3(100)}{8} = 3.75 and x_2 = \frac{0.7(100)}{2} = 35$

Expenditure on x_1 and x_2 is $\frac{p_1x_1}{M} = 0.3(exponentof x_1) and x_2 = \frac{p_2x_2}{M} 0.7(exp\,o\,nentof x_2)$ respectively. $U_0 = (3.75)^{0.3}(35)^{0.7} = 17.9(Figure4)$. The final bundle of goods after falling in p_1 from £8 to £4 (=p_1^*) is $x_1 = \frac{0.3(100)}{4} = 12.5 and x_2 = \frac{0.7(100)}{2} = 35$ Thus, the final bundle at G (Figure 3) is (7.5, 35).

The utility level is $U_2 = (7.5)^{0.3}(35)^{0.7} = 22.03(Figure4)$. The utility level at Hicks bundle is $U_0 = 17.89(Figure3and4)$ The utility maximization condition is $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2} \Rightarrow \frac{0.3x_1^{-0.7}x_2^{0.7}}{4} = \frac{0.7x_1^{0.3}x_2^{-0.3}}{2}$ $\Rightarrow x_2 = 4.66x_1$. Substituting this in the utility function $U = x_1^{0.3}x^{0.7} \Rightarrow 17.9 = x_1^{0.3}(4.66x_1)^{0.7}$ $x_1 = 6.10, x_2 = 28.42$.

The Hicks Demand functions are $x_1^H = U\left(\frac{3p_2}{7p_1^*}\right)^{0.7}, x_2^H = U\left(\frac{7p_1^*}{3p_2}\right)^{0.3}$ (5)

$\Rightarrow x_1^H = 17.9\left(\frac{3(2)}{7(4)}\right)^{0.7} = 6.08 and x_2^H = 17.9\left(\frac{7(4)}{3(2)}\right)^{0.3} = 28.41$. The Hicks demand functions at tangency point F1 in Figure 3 are (6.08, 28.41).

The Slutsky demand functions are $x_1^S = \frac{0.3M^*}{p_1^*} and, x_2^S = \frac{0.7M^*}{p_2}$ (6)

Plugging in the respective values, we get the Slutsky bundle $x_1^S = \frac{0.3(85)}{4} = 6.37, x_2^S = \frac{0.7(85)}{2} = 29.75$. at B Figure 4. It is to be noted that M^*= Minimum income required to buy the initial bundle at a decreased price of x_1 denoted by p_1^*=£4. M^*= $p_1^*x_1+p_2x_2$= 4(3.75) +2(35)=85.The utility level at the Slutsky bundle is U_1= $(6.37)^{0.3}(29.75)^{0.7}$= 18.73 Figure 4. The utility function $U = x_1^{0.3}x_2^{0.7}$, holds the following property.

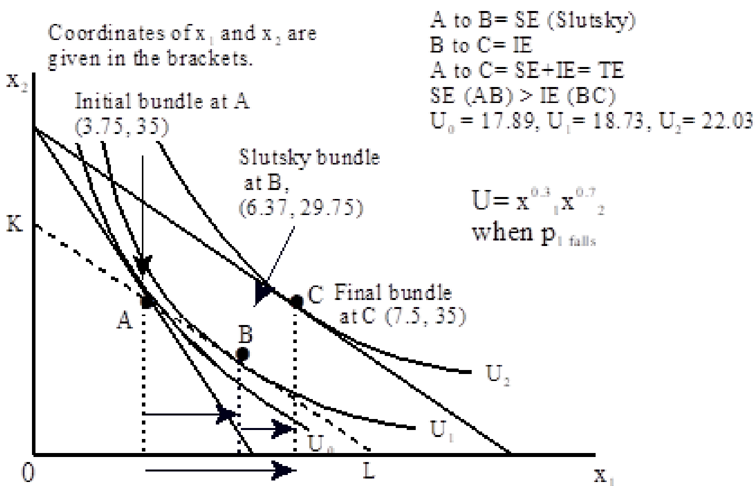

Figure 4: Slutsky Substitution Effect and Income Effect

$$\frac{\%expenditureonx_1}{\%expenditureonx_2}=\frac{utilityelaticityofx_1}{utilityelasticityofx_2}=\frac{exponentofx_1(0.3)}{exp\ o\ nentofx_2(0.7)}=\frac{IE(1.13)}{SE(2.62)}=0.43$$
$$IE=0.43SE, SEisbiggerthanIE. TE=SE(2.62)+IE(1.13)=3.75(Slutskyapproach)$$

In this case, the SE is bigger than the IE. When expenditure incurred on good x_1 was less than the good x_2, falling in its price p_1 the magnitude of SE was greater than IE.

An increase in consumption of good $x_1(\Delta x_1)$ due to the Hicks SE and IE was 2.34 and 1.41 respectively (Figure 3). An increase in consumption of good $x_1(\Delta x_1)$ due to the Slutsky SE and IE was 2.62 and 1.13 respectively (Figure 4). The magnitude of SE and IE are directly related to the proportion of income spent on each good which is represented by the exponents of the goods in the given utility function. The SE was greater than the IE under both approaches.

For $U=x_1^{0.7}x^3$, the Marshallian demand functions are $x_1=\frac{0.7M}{p_1}andx_2=\frac{0.3M}{p_2}$ (7)
The Initial bundle of goods at F (Fig.3) is $x_1=\frac{0.7(100)}{8}=8.75andx_2=\frac{0.3(100)}{2}=15$

Expenditure on x_1 and x_2 is $\frac{p_1x_1}{M}=0.7(exponentofx_1)andx_2=\frac{p_2x_2}{M}0.3(exponentofx_2)$ respectively. $U_0=(8.75)^{0.7}(15)^{0.3}=10.28(Figure5and6)$. The final bundle of goods after falling in p_1 from £8 to £4 (=p_1^*) is $x_1=\frac{0.7(100)}{4}=17.5andx_2=\frac{0.3(100)}{2}=15$ Thus, the final bundle at C (Figure 5 and 6) is (17.5, 15).

The utility level is $U_2=(17.5)^{0.7}(15)^{0.3}=16.70(Figure6)$. The utility level at Hicks bundle is $U_0=10.28(Figure5and6), andU_2=(17.5)^{0.7}(15)^{0.3}=16.70(Figure6)$

The utility maximization condition is $\frac{MU_{x_1}}{p_1}=\frac{MU_{x_2}}{p_2}\Rightarrow\frac{0.7x_1^{-0.3}x_2^{0.3}}{4}=\frac{o.3x_1^{0.7}x_2^{-0.7}}{2}\Rightarrow$
$x_2=0.85x_1$. $U=x_1^{0.7}{x_2}^{0.3}\Rightarrow 10.28=x_1^{0.7}(0.85x_1)^{0.3}\Rightarrow x_1=10.79andx_2=9.17$
$x_1^H=U\left(\frac{7p_2}{3p_1^*}\right)^{0.3}, x_2^H=U\left(\frac{3p_1^*}{7p_2}\right)^{0.7}$
$$\Rightarrow x_1^H=17.9\left(\frac{3(2)}{7(4)}\right)^{0.7}=10.77andx_2^H=17.9\left(\frac{7(4)}{3(2)}\right)^{0.3}=9.24 \quad (8)$$

The Hicks demand functions at tangency point B in Figure 5 are (10.77, 9.24).

The Slutsky demand functions are $x_1^S = \frac{0.7M^*}{p_1^*} and, x_2^S = \frac{0.3M^*}{p_2}$ (9) plugging in the respective values, we get the Slutsky bundle $x_1^S = \frac{0.7(65)}{4} = 11.37, x_2^S = \frac{0.3(65)}{2} = 9.75$. at B Figure 6. It is to be noted that M^*= Minimum income required to buy the initial bundle at a decreased price of x_1 which is denoted by p_1^*=£4. $M^* = p_1^* x_1 + p_2 x_2$= 4(8.74) +2(15)=65.The utility level at the Slutsky bundle is $U_1 = (11.37)^{0.7}(9.75)^{0.3} = 10.76$ Figure 6. The utility function $U = x_1^{0.7} x_2^{0.3}$, holds the following property.

$$\frac{\%expenditureonx_1}{\%expenditureonx_2} = \frac{utilityelaticityof x_1}{utilityelasticityof x_2} = \frac{exponentof x_1(0.7)}{exponentof x_2(0.3)} = \frac{IE(6.13)}{SE(2.62)} = 2.33$$
$$IE = 2.33SE, theIEisbiggerthantheSEandTE = SE + IE = 8.85$$

In this case, the IE is bigger than the SE. When expenditure incurred on good x_1 was greater than the good x_2, a decrease in price p_1 shows that the magnitude of SE was greater than IE.

An increase in consumption of good $x_1(\Delta x_1)$ due to the Hicks SE and IE was 2.02 and 61.41 respectively (Figure 5). An increase in good $x_1(\Delta x_1)$ consumption due to the Slutsky SE and IE was 2.62 and 6.13 respectively (Figure 6). The magnitude of SE and IE are directly related to the proportion of income spent on each good represented by the exponents of the goods in the given utility function. The SE was greater than the IE under both approaches.

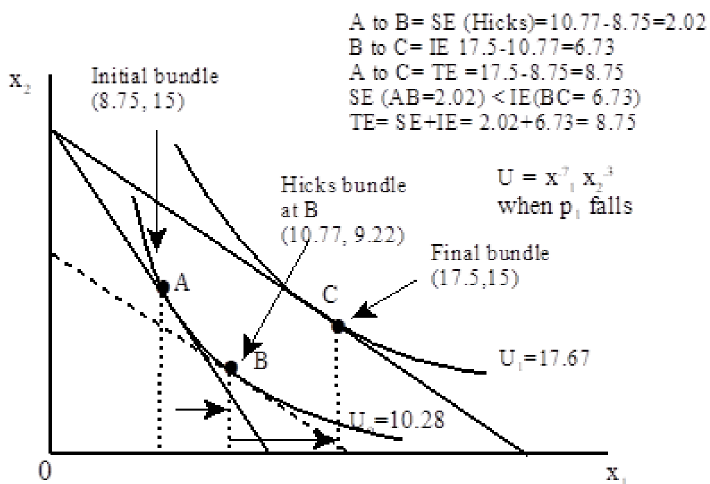

Figure 5: Hicks Substitution Effect and Income Effect

$$\frac{\%expenditureonx_1}{\%expenditureonx_2} = \frac{utilityelasticityof x_1}{utilityelasticityof x_2} = \frac{Exponentof x_1}{Exponentof x_2} = \frac{IE(6.73)}{SE(2.02)} = 3.33$$
$$\Rightarrow IE = 3.33SE, IE > SETE = SE(2.02) + IE(6.73) = 8.75(Hicksapproach)$$

The proportion of the income effect of a price change depends on the income elasticity of demand and what percentage of the budget is being spent on that good. When a small percentage of the budget is spent on the good whose price changes, the substitution effect will be relatively bigger. The converse holds when the price of a good changes on which a bigger proportion of income is spent.

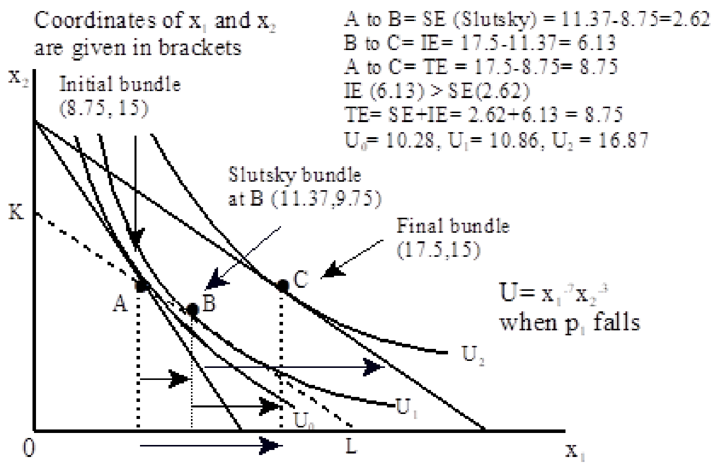

Figure 6: Slutsky Substitution Effect and Income Effect

$$\frac{\%expenditureonx_1}{\%expenditureonx_2} = \frac{utilityelasticityofx_1}{utilityelasticityofx_2} = \frac{Exponentofx_1(.7)}{Exponentofx_2(.3)} = \frac{IE(6.13)}{SE(2.62)} = 2.33 \Rightarrow$$
$$IE = 2.33SE, IE > SE, TE = SE(2.62) + IE(6.13) = 8.85(Slutskyapproach)$$

Since each consumer’s response to a price change depends on the sizes of the substitution and income effects, these effects play a role in determining the price elasticity of demand. *Ceteris paribus*, the larger the substitution effect, the greater the absolute value of the price elasticity of demand (Table 2). When the income effect moves in the same direction as the substitution effect, a greater income effect also contributes to a greater price elasticity of demand. There are; cases in which the substitution and income effects move in opposite directions, such as an inferior good and a Giffen good.

Table 1

Hicks’s Breakdown of The Total Effect of a Fall in The Price p_1Of Good x_1 Into The Substitution Effect and Income Effec

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial h_1}{\partial p_1} + (-x_1\frac{\partial x_1}{\partial M})$$
$$TE = SE + IE$$

Nature of good x_1	Substitution Effect $\frac{\partial h_1}{\partial p_1}$ (Direction of change)	Income Effect $-x_1\frac{\partial x_1}{\partial M}$ (Direction of change)	Which is bigger SE. or IE (Magnitude of Change) when $p_1$falls	Price effect on quantity demanded of x_1. $\frac{\partial x_1}{\partial p_1}$
Normal good $\frac{\partial x_1}{\partial M}> 0$ $\frac{\partial x_1}{\partial p_1} < 0$	Increase in quantity demanded of x_1 $\frac{\partial h_1}{\partial p_1}> 0$ (+ ve effect)	Increase in quantity demanded of x_1 (+ ve effect)	$ForU = x_1^5x_2^5 SE = IE$ $ForU = x_1^3x_2^7 SE > IE$ $ForU = x_1^7x_2^3 IE > SE$ Positive I.E. and S.E. reinforce each other. $U = x_1^\alpha x_2^\beta where \alpha + \beta = 1$ $subjectto M = p_1x_1 + p_2x_2$	Positive: p_1 and quantity demanded of $x_1$move in the opposite direction
Inferior good $\frac{\partial x_1}{\partial M}< 0$ $\frac{\partial x_1}{\partial p_1} < 0$	Increase in quantity demanded of x_1 $\frac{\partial h_1}{\partial p_1} > 0$ (+ ve effect)	Decrease in quantity demanded of x_1 (- ve effect)	$SE > IE$ $\frac{\partial h_1}{\partial p_1}$ (+ve) > $-x_1\frac{\partial x_1}{\partial M}$ (-ve) Positive SE outweighs negative IE	Positive: p_1 and quantity demanded of $x_1$move in the opposite direction
Giffen good $\frac{\partial x_1}{\partial p_1} > 0,$ $\frac{\partial x_1}{\partial M}<< 0$	Increase in quantity demanded of x_1 $\frac{\partial h_1}{\partial p_1} > 0$ (+ ve effect)	Decrease in quantity demanded of x_1 (-ve effect)	$SE < IE$ $\frac{\partial h_1}{\partial p_1}$ (+ve) < $-x_1\frac{\partial x_1}{\partial M}$ (-ve) Negative I.E. outweighs positive S.E.	Negative: p_1 and quantity demanded of $x_1$move in the same direction

Table 2
Comparison of price elasticity of demand (ε) of the Marshallian, the Slutsky and the Hicks demand curves, $U=X_1^{.5}X_2^{.5}$ (fall in p_1)

	The initial quantity demanded of X_1 at F in Fig.1 before fall in its price	The Marshallian quantity demanded of X_1 after a fall in its price from £8 to £4, $p_1^*= 4$ M=100	The Hicks's quantity demanded of X_1 after a fall in its price from £8 to £4, $p_1^*= 4$, $p_2=2$ U= 12.5	The Slutsky's quantity demanded of X_1 after a fall in its price from £8 to £4, $p_1^*= 4$, $M^* =75$
	6.25	The Marshallian $X_1 = \frac{0.5M}{p_1^*}$ $X_1=12.5$	The Hicks $X_1 = U\sqrt{\frac{p_2}{p_1^*}}$ $X_1 = 8.83$	The Slutsky $X_1 = \frac{0.5M^*}{p_1^*}$ $X_1 = 9.37$
% Δ in quantity		100%	40%	49.92%
% Δ in price	100%		100%	100%
The elasticity of demand = ε	$\varepsilon = \frac{\%\Delta in quantity}{\%\Delta in price} = 1$		$\varepsilon = 0.40$	$\varepsilon = 0.49$

The price elasticity of demand of the Marshallian demand curve is unity, Slutsky's is 0.49 and that of the Hicks's is 0.40. Figure 7 is the graphical exposition of this.

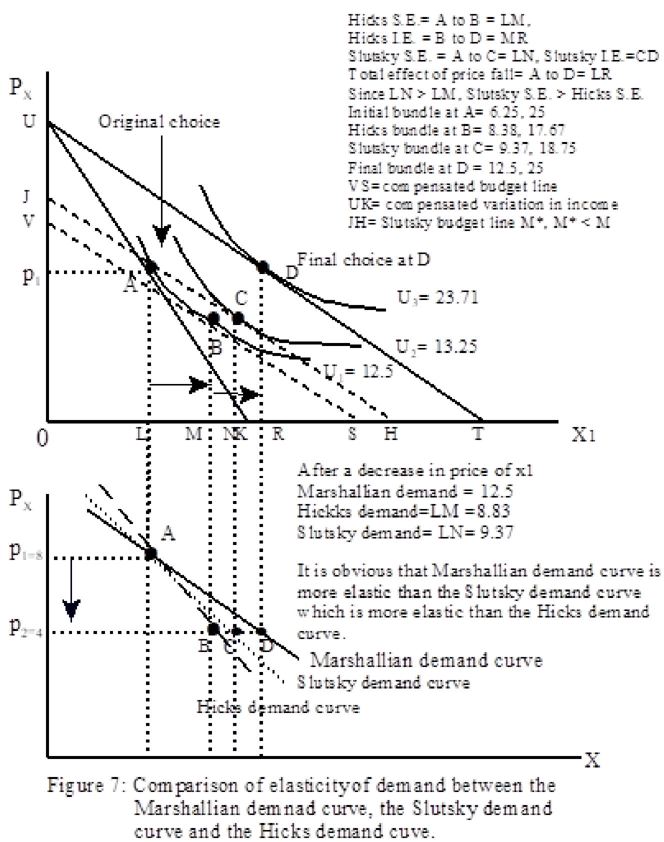

Figure 7: Comparison of Elasticity of Demand between the Marshallian Demand Curve, the Slutsky Demand Curve and the Hicks Demand Curve

4. Conclusion

Our research explored the link between the exponents of the utility function and the size of the substitution effect and the income effect in the Slutsky equation. The two effects are equal when the amount spent on each good is the same, i.e. exponents of both goods in the utility function representing the proportion of income spent on them are equal. The substitution effect is bigger than the income effect when the price of good 1 decrease and the proportion of income spent on it is less than that of good 2. The income effect is bigger than the substitution effect when the price of good 1 decrease and the proportion of income $\frac{p_1 x_1}{M}$ spent on it is more than good 2. The comparison shows that the total effect of price change on the quantity demanded is greatest when the proportion of income spent on it is greater than the other good; because the I.E. is larger than the S.E. The converse holds.

It is also to be noted that for all three utility functions discussed above, $\frac{\%expenditureonx_1}{\%expenditureonx_2} = \frac{utilityelasticityofx_1}{utilityelasticityofx_2} = \frac{Exponentofx_1}{Exponentofx_2} = \frac{IE}{SE}$ It has also been shown that the Marshallian demand curve is more price elastic than the Slutsky demand curve which is more elastic than the Hicks demand curve. Our research will meet a hitherto unmet need to understand this topic.

Authors' Contribution

Parvez Azim: conceptualized the study, did calculations, and drafted the initial manuscript
Shabbir Ahmad Gondal: reviewed and edited the manuscript for critical insights
Azka Amin: contributed to the literature review, did proofreading, and gave suggestions to improve the initial draft.

Conflict of Interests/Disclosures

The authors declared no potential conflict of interest w.r.t the article's research, authorship and/or publication.

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